



MARKSCHEME

May 2014

MATHEMATICS

Higher Level

Paper 2

22 pages

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to Scoris instructions and the document “**Mathematics HL: Guidance for e-marking May 2014**”. It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the ‘must be seen’ marks), use the ticks with numbers to stamp full marks
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by Scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **AI**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, for example, **MIAI**, this usually means **MI** for an **attempt** to use an appropriate method (for example, substitution into a formula) and **AI** for using the **correct** values.
- Where the markscheme specifies **(M2)**, **N3**, *etc*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 ***N* marks**

Award *N* marks for **correct** answers where there is **no** working.

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 **Implied marks**

Implied marks appear in **brackets**, for example, (**MI**), and can only be awarded if **correct** work is seen or implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 **Follow through marks**

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (for example, $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 **Mis-read**

If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (for example, $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).

7 **Discretionary marks (*d*)**

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation **DM** should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = (2\cos(5x-3))5 \quad (=10\cos(5x-3)) \quad \text{AI}$$

Award **AI** for $(2\cos(5x-3))5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

SECTION A

1. (a) (i) $n = 27$ (A1)

METHOD 1

$$S_{27} = \frac{14+196}{2} \times 27 \quad (M1)$$

$$= 2835 \quad A1$$

METHOD 2

$$S_{27} = \frac{27}{2}(2 \times 14 + 26 \times 7) \quad (M1)$$

$$= 2835 \quad A1$$

METHOD 3

$$S_{27} = \sum_{n=1}^{27} 7 + 7n \quad (M1)$$

$$= 2835 \quad A1$$

(ii) $\sum_{n=1}^{27} (7 + 7n)$ or equivalent A1

Note: Accept $\sum_{n=2}^{28} 7n$

[4 marks]

(b) $\frac{n}{2}(2000 - 6(n-1)) < 0$ (M1)

$$n > 334.333$$

$$n = 335 \quad A1$$

Note: Accept working with equalities.

[2 marks]

Total [6 marks]

2. (a) **METHOD 1**

$$\mu = \frac{1}{2} \times (17.1 + 21.3) \quad (M1)$$

$$\mu = 19.2 \text{ (kg)} \quad A1$$

finding z value for the upper quartile = 0.674489K

$$0.674489K = \frac{21.3 - 19.2}{\sigma} \text{ or } -0.674489K = \frac{17.1 - 19.2}{\sigma} \quad M1$$

$$\sigma = 3.11 \text{ (kg)} \quad A1$$

METHOD 2

finding z value for the upper quartile = 0.674489K

from symmetry the z value for a lower quartile is $-0.674489K$ M1

forming two simultaneous equations:

$$-0.674489K = \frac{17.1 - \mu}{\sigma}$$

$$0.674489K = \frac{21.3 - \mu}{\sigma} \quad M1$$

solving gives:

$$\mu = 19.2 \text{ (kg)} \quad A1$$

$$\sigma = 3.11 \text{ (kg)} \quad A1$$

[4 marks]

(b) using $100 \times P(X > 22) = 100 \times 0.184241K$
 $= 18$ A1

Note: Accept 18.4

[1 mark]

Total [5 marks]

3. (a) $x_A = 2.87$ *A1*
 $x_B = 6.78$ *A1*
[2 marks]

(b) $\int_{2.87172K}^{6.77681K} 1 - 2\sin x - x^2 e^{-x} \, dx$ *(M1)(A1)*
 $= 6.76$ *A1*

Note: Award *(M1)* for definite integral and *(A1)* for a correct definite integral.

[3 marks]

Total [5 marks]

4. (a) **METHOD 1**
 $2\arcsin\left(\frac{1.5}{4}\right)$ *M1*
 $\alpha = 0.769^\circ$ (44.0°) *A1*

METHOD 2

using the cosine rule:

$3^2 = 4^2 + 4^2 - 2(4)(4)\cos\alpha$ *M1*
 $\alpha = 0.769^\circ$ (44.0°) *A1*

[2 marks]

- (b) one segment
 $A_1 = \frac{1}{2} \times 4^2 \times 0.76879 - \frac{1}{2} \times 4^2 \times \sin(0.76879)$ *M1A1*
 $= 0.58819K$ *(A1)*
 $2A_1 = 1.18 \text{ (cm}^2\text{)}$ *A1*

Note: Award *M1* only if both sector and triangle are considered.

[4 marks]

Total [6 marks]

5. expanding $(x-1)^3 = x^3 - 3x^2 + 3x - 1$ *AI*

expanding $\left(\frac{1}{x} + 2x\right)^6$ gives

$$64x^6 + 192x^4 + 240x^2 + \frac{60}{x^2} + \frac{12}{x^4} + \frac{1}{x^6} + 160$$
(M1)AI AI

Note: Award *(M1)* for an attempt at expanding using binomial.

Award *AI* for $\frac{60}{x^2}$.

Award *AI* for $\frac{12}{x^4}$.

$$\frac{60}{x^2} \times -1 + \frac{12}{x^4} \times -3x^2$$
(M1)

Note: Award *(M1)* only if both terms are considered.

therefore coefficient x^{-2} is -96 *AI*

Note: Accept $-96x^{-2}$

Note: Award full marks if working with the required terms only without giving the entire expansion.

[6 marks]

6. (a) (i) $0.6^3 \times 0.4^3$ *(M1)*

Note: Award *(M1)* for use of the product of probabilities.

$$= 0.0138$$
AI

(ii) binomial distribution $X : B(6, 0.6)$ *(M1)*

Note: Award *(M1)* for recognizing the binomial distribution.

$$P(X = 3) = {}^6C_3 (0.6)^3 (0.4)^3$$

$$= 0.276$$
AI

Note: Award *(M1)AI* for ${}^6C_3 \times 0.0138 = 0.276$.

[4 marks]

continued...

Question 6 continued

- (b) $Y : B(n, 0.4)$
 $P(Y \geq 1) > 0.995$
 $1 - P(Y = 0) > 0.995$
 $P(Y = 0) < 0.005$ **(M1)**

Note: Award **(M1)** for any of the last three lines. Accept equalities.

$0.6^n < 0.005$ **(M1)**

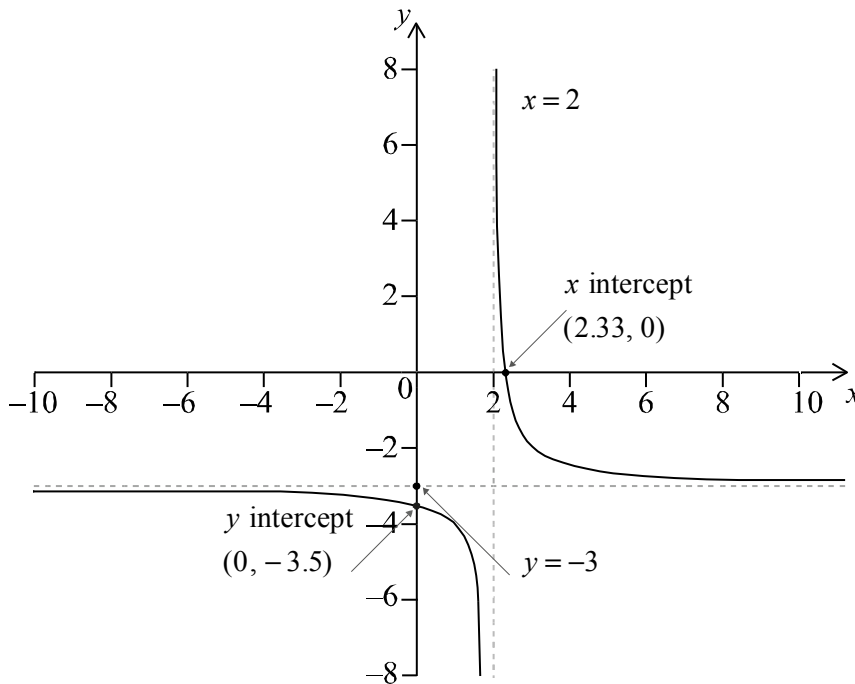
Note: Award **(M1)** for attempting to solve $0.6^n < 0.005$ using any method, eg, logs, graphically, use of solver. Accept an equality.

$n > 10.4$
 $\therefore n = 11$ **A1**

[3 marks]

Total [7 marks]

7. (a)



A1A1A1

Note: Award **A1** for correct shape, **A1** for $x = 2$ clearly stated and **A1** for $y = -3$ clearly stated.

x intercept $(2.33, 0)$ and y intercept $(0, -3.5)$ **A1**

Note: Accept -3.5 and 2.33 ($7/3$) marked on the correct axes.

[4 marks]
 continued...

Question 7 continued

(b) $x = -3 + \frac{1}{y-2}$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$$x + 3 = \frac{1}{y - 2}$$

$$y - 2 = \frac{1}{x + 3}$$
M1

Note: Award **M1** for attempting to make y the subject.

$$f^{-1}(x) = 2 + \frac{1}{x+3} \left(= \frac{2x+7}{x+3} \right), x \neq -3$$
A1A1

Note: Award **A1** only if $f^{-1}(x)$ is seen. Award **A1** for the domain.

[4 marks]

Total [8 marks]

8. (a) $\frac{\mu^2 e^{-\mu}}{2!} + \frac{\mu^3 e^{-\mu}}{3!} = \frac{\mu^5 e^{-\mu}}{5!}$ **(M1)**

$$\frac{\mu^2}{2} + \frac{\mu^3}{6} - \frac{\mu^5}{120} = 0$$

$$\mu = 5.55$$
A1

[2 marks]

(b) $\sigma = \sqrt{5.55\dots} = 2.35598\dots$ **(M1)**
 $P(3.19 \leq X \leq 7.9)$
 $P(4 \leq X \leq 7)$
 $= 0.607$ **A1**

[2 marks]

Total [4 marks]

9. **METHOD 1**

volume of a cone is $V = \frac{1}{3}\pi r^2 h$

given $h = r$, $V = \frac{1}{3}\pi h^3$ **M1**

$\frac{dV}{dh} = \pi h^2$ **(A1)**

when $h = 4$, $\frac{dV}{dt} = \pi \times 4^2 \times 0.5$ (using $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$) **M1A1**

$\frac{dV}{dt} = 8\pi$ (= 25.1) (cm³ min⁻¹) **A1**

METHOD 2

volume of a cone is $V = \frac{1}{3}\pi r^2 h$

given $h = r$, $V = \frac{1}{3}\pi h^3$ **M1**

$\frac{dV}{dt} = \frac{1}{3}\pi \times 3h^2 \times \frac{dh}{dt}$ **A1**

when $h = 4$, $\frac{dV}{dt} = \pi \times 4^2 \times 0.5$ **M1A1**

$\frac{dV}{dt} = 8\pi$ (= 25.1) (cm³ min⁻¹) **A1**

METHOD 3

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi \left(2r h \frac{dr}{dt} + r^2 \frac{dh}{dt} \right)$ **M1A1**

Note: Award **M1** for attempted implicit differentiation and **A1** for each correct term on the RHS.

when $h = 4$, $r = 4$, $\frac{dV}{dt} = \frac{1}{3}\pi (2 \times 4 \times 4 \times 0.5 + 4^2 \times 0.5)$ **M1A1**

$\frac{dV}{dt} = 8\pi$ (= 25.1) (cm³ min⁻¹) **A1**

[5 marks]

10. (a) **METHOD 1**

expanding the brackets first:

$$x^4 + 2x^2y^2 + y^4 = 4xy^2$$

M1

$$4x^3 + 4xy^2 + 4x^2y \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 4y^2 + 8xy \frac{dy}{dx}$$

M1A1A1

Note: Award *M1* for an attempt at implicit differentiation.
Award *A1* for each side correct.

$$\frac{dy}{dx} = \frac{-x^3 - xy^2 + y^2}{yx^2 - 2xy + y^3} \text{ or equivalent}$$

A1

METHOD 2

$$2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) = 4y^2 + 8xy \frac{dy}{dx}$$

M1A1A1

Note: Award *M1* for an attempt at implicit differentiation.
Award *A1* for each side correct.

$$(x^2 + y^2) \left(x + y \frac{dy}{dx} \right) = y^2 + 2xy \frac{dy}{dx}$$

$$x^3 + x^2y \frac{dy}{dx} + y^2x + y^3 \frac{dy}{dx} = y^2 + 2xy \frac{dy}{dx}$$

M1

$$\frac{dy}{dx} = \frac{-x^3 - xy^2 + y^2}{yx^2 - 2xy + y^3} \text{ or equivalent}$$

A1

[5 marks]

(b) **METHOD 1**

at (1, 1), $\frac{dy}{dx}$ is undefined

M1A1

$$y = 1$$

A1

METHOD 2

$$\text{gradient of normal} = -\frac{1}{\frac{dy}{dx}} = -\frac{(yx^2 - 2xy + y^3)}{(-x^3 - xy^2 + y^2)}$$

M1

at (1, 1) gradient = 0

A1

$$y = 1$$

A1

[3 marks]

Total [8 marks]

SECTION B

11. (a) $a \int_0^{\frac{\pi}{2}} x \cos x \, dx = 1$ **(M1)**

integrating by parts:

$u = x \quad v' = \cos x$ **M1**

$u' = 1 \quad v = \sin x$

$\int x \cos x \, dx = x \sin x + \cos x$ **A1**

$[x \sin x + \cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$ **A1**

$a = \frac{1}{\frac{\pi}{2} - 1}$ **A1**

$= \frac{2}{\pi - 2}$ **AG**

[5 marks]

(b) $P\left(X < \frac{\pi}{4}\right) = \frac{2}{\pi - 2} \int_0^{\frac{\pi}{4}} x \cos x \, dx = 0.460$ **(M1)A1**

Note: Accept $\frac{2}{\pi - 2} \left(= \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1 \right)$ or equivalent

[2 marks]

(c) (i) mode = 0.860 **A1**
 (x-value of a maximum on the graph over the given domain)

(ii) $\frac{2}{\pi - 2} \int_0^m x \cos x \, dx = 0.5$ **(M1)**

$\int_0^m x \cos x \, dx = \frac{\pi - 2}{4}$

$m \sin m + \cos m - 1 = \frac{\pi - 2}{4}$ **(M1)**

median = 0.826 **A1**

Note: Do not accept answers containing additional solutions.

[4 marks]

continued...

Question 11 continued

$$(d) \quad P\left(X < \frac{\pi}{8} \mid X < \frac{\pi}{4}\right) = \frac{P\left(X < \frac{\pi}{8}\right)}{P\left(X < \frac{\pi}{4}\right)} \quad \text{M1}$$

$$= \frac{0.129912}{0.459826}$$

$$= 0.283 \quad \text{A1}$$

[2 marks]

Total [13 marks]

12. (a) $C = AX \times 5k + XB \times k$ (M1)

Note: Award (M1) for attempting to express the cost in terms of AX, XB and k.

$$= 5k\sqrt{450^2 + x^2} + (1000 - x)k \quad \text{A1}$$

$$= 5k\sqrt{202500 + x^2} + (1000 - x)k \quad \text{AG}$$

[2 marks]

(b) (i) $\frac{dC}{dx} = k \left[\frac{5 \times 2x}{2\sqrt{202500 + x^2}} - 1 \right] = k \left(\frac{5x}{\sqrt{202500 + x^2}} - 1 \right)$ M1A1

Note: Award M1 for an attempt to differentiate and A1 for the correct derivative.

continued...

Question 12 continued

(ii) attempting to solve $\frac{dC}{dx} = 0$ **MI**

$$\frac{5x}{\sqrt{202500 + x^2}} = 1$$
 (A1)

$$x = 91.9 \text{ (m)} \left(= \frac{75\sqrt{6}}{2} \text{ (m)} \right)$$
 A1

METHOD 1

for example,

at $x = 91$ $\frac{dC}{dx} = -0.00895k < 0$ **MI**

at $x = 92$ $\frac{dC}{dx} = 0.001506k > 0$ **A1**

Note: Award **MI** for attempting to find the gradient either side of $x = 91.9$ and **A1** for two correct values.

thus $x = 91.9$ gives a minimum **AG**

METHOD 2

$$\frac{d^2C}{dx^2} = \frac{1012500k}{(x^2 + 202500)^{\frac{3}{2}}}$$

at $x = 91.9$ $\frac{d^2C}{dx^2} = 0.010451k > 0$ **(M1)A1**

Note: Award **MI** for attempting to find the second derivative and **A1** for the correct value.

Note: If $\frac{d^2C}{dx^2}$ is obtained and its value at $x = 91.9$ is not calculated, award **(M1)A1** for correct reasoning *eg*, both numerator and denominator are positive at $x = 91.9$.

thus $x = 91.9$ gives a minimum **AG**

METHOD 3

Sketching the graph of either C versus x or $\frac{dC}{dx}$ versus x . **MI**

Clearly indicating that $x = 91.9$ gives the minimum on their graph. **A1**

[7 marks]

continued...

Question 12 continued

(c) $C_{\min} = 3205 k$ **A1**

Note: Accept 3200k.
Accept 3204 k.

[1 mark]

(d) $\arctan\left(\frac{450}{91.855865K}\right) = 78.463K^\circ$ **M1**

$180 - 78.463K = 101.537K$
 $= 102^\circ$

A1

[2 marks]

(e) (i) when $\theta = 120^\circ$, $x = 260(m) \left(\frac{450}{\sqrt{3}}(m)\right)$ **A1**

(ii) $\frac{133.728K}{3204.5407685K} \times 100\%$ **M1**

$= 4.17(\%)$ **A1**

[3 marks]

Total [15 marks]

13. (a) let $P(n)$ be the proposition $z^n = r^n(\cos n\theta + i\sin n\theta)$, $n \in \mathbb{C}^+$
 let $n = 1 \Rightarrow$
 LHS = $r(\cos\theta + i\sin\theta)$
 RHS = $r(\cos\theta + i\sin\theta)$, $\therefore P(1)$ is true **R1**
 assume true for $n = k \Rightarrow r^k(\cos\theta + i\sin\theta)^k = r^k(\cos(k\theta) + i\sin(k\theta))$ **M1**

Note: Only award the **M1** if truth is assumed.

- now show $n = k$ true implies $n = k + 1$ also true
 $r^{k+1}(\cos\theta + i\sin\theta)^{k+1} = r^{k+1}(\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)$ **M1**
 $= r^{k+1}(\cos(k\theta) + i\sin(k\theta))(\cos\theta + i\sin\theta)$
 $= r^{k+1}(\cos(k\theta)\cos\theta - \sin(k\theta)\sin\theta + i(\sin(k\theta)\cos\theta + \cos(k\theta)\sin\theta))$ **A1**
 $= r^{k+1}(\cos(k\theta + \theta) + i\sin(k\theta + \theta))$ **A1**
 $= r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta) \Rightarrow n = k + 1$ is true **A1**
 $P(k)$ true implies $P(k + 1)$ true and $P(1)$ is true, therefore by mathematical induction statement is true for $n \geq 1$ **R1**

Note: Only award the final **R1** if the first 4 marks have been awarded.

[7 marks]

- (b) (i) $u = 2\text{cis}\left(\frac{\pi}{3}\right)$ **A1**
 $v = \sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right)$ **A1**

Notes: Accept 3 sf answers only. Accept equivalent forms.
 Accept $2e^{\frac{\pi}{3}i}$ and $\sqrt{2}e^{-\frac{\pi}{4}i}$.

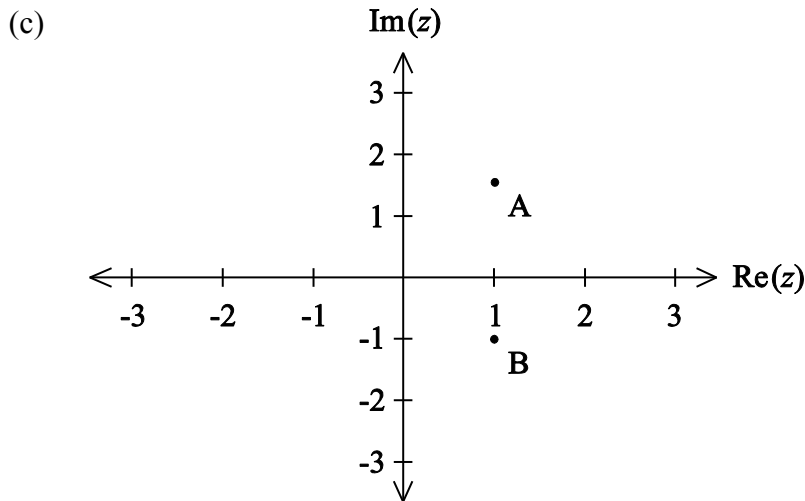
- (ii) $u^3 = 2^3\text{cis}(\pi) = -8$
 $v^4 = 4\text{cis}(-\pi) = -4$ **(M1)**
 $u^3v^4 = 32$ **A1**

Notes: Award **(M1)** for an attempt to find u^3 and v^4 .
 Accept equivalent forms.

[4 marks]

continued...

Question 13 continued



A1

Note: Award **A1** if A or $1 + \sqrt{3}i$ and B or $1 - i$ are in their correct quadrants, are aligned vertically and it is clear that $|u| > |v|$.

[1 mark]

(d) Area = $\frac{1}{2} \times 2 \times \sqrt{2} \times \sin\left(\frac{5\pi}{12}\right)$
 $= 1.37 \left(= \frac{\sqrt{2}}{4} (\sqrt{6} + \sqrt{2}) \right)$

M1A1

A1

Notes: Award *M1A0A0* for using $\frac{7\pi}{12}$.

[3 marks]

(e) $(z - 1 + i)(z - 1 - i) = z^2 - 2z + 2$

M1A1

Note: Award *M1* for recognition that a complex conjugate is also a root.

$$(z - 1 - \sqrt{3}i)(z - 1 + \sqrt{3}i) = z^2 - 2z + 4$$

A1

$$(z^2 - 2z + 2)(z^2 - 2z + 4) = z^4 - 4z^3 + 10z^2 - 12z + 8$$

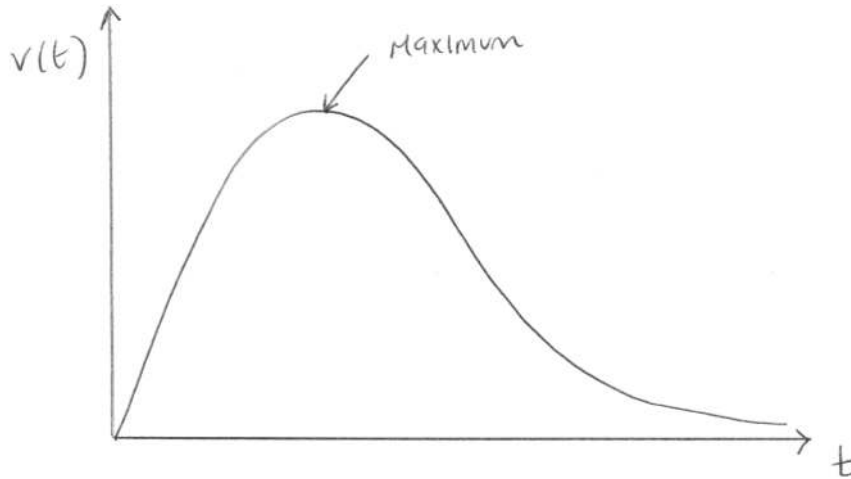
M1A1

Note: Award *M1* for an attempt to expand two quadratics.

[5 marks]

Total [20 marks]

14. (a)



A1

A1 for correct shape and correct domain

$$(1.41, 0.0884) \left(\sqrt{2}, \frac{\sqrt{2}}{16} \right)$$

A1

[2 marks]

(b) **EITHER**

$$u = t^2$$

$$\frac{du}{dt} = 2t$$

A1

OR

$$t = u^{\frac{1}{2}}$$

$$\frac{dt}{du} = \frac{1}{2} u^{-\frac{1}{2}}$$

A1

THEN

$$\int \frac{t}{12+t^4} dt = \frac{1}{2} \int \frac{du}{12+u^2}$$

M1

$$= \frac{1}{2\sqrt{12}} \arctan \left(\frac{u}{\sqrt{12}} \right) (+c)$$

M1

$$= \frac{1}{4\sqrt{3}} \arctan \left(\frac{t^2}{2\sqrt{3}} \right) (+c) \text{ or equivalent}$$

A1

[4 marks]

continued...

Question 14 continued

$$\begin{aligned}
 \text{(c)} \quad & \int_0^6 \frac{t}{12+t^4} dt && \text{(M1)} \\
 & = \left[\frac{1}{4\sqrt{3}} \arctan\left(\frac{t^2}{2\sqrt{3}}\right) \right]_0^6 && \text{M1} \\
 & = \frac{1}{4\sqrt{3}} \left(\arctan\left(\frac{36}{2\sqrt{3}}\right) \right) \left(= \frac{1}{4\sqrt{3}} \left(\arctan\left(\frac{18}{\sqrt{3}}\right) \right) \right) \text{(m)} && \text{A1}
 \end{aligned}$$

Note: Accept $\frac{\sqrt{3}}{12} \arctan(6\sqrt{3})$ or equivalent.

[3 marks]

$$\text{(d)} \quad \frac{dv}{ds} = \frac{1}{2\sqrt{s(1-s)}} \quad \text{(A1)}$$

$$a = v \frac{dv}{ds}$$

$$a = \arcsin(\sqrt{s}) \times \frac{1}{2\sqrt{s(1-s)}} \quad \text{(M1)}$$

$$a = \arcsin(\sqrt{0.1}) \times \frac{1}{2\sqrt{0.1 \times 0.9}}$$

$$a = 0.536 \text{ (ms}^{-2}\text{)} \quad \text{A1}$$

[3 marks]

Total [12 marks]